Exam. Code : 211002
Subject Code: 4897

M.Sc. (Mathematics) 2nd Semester (Batch 2021-23)

REAL ANALYSIS-II

Paper—Math-561

Time Allowed—3 Hours] [Maximum Marks—100

Note :— Attempt **FIVE** questions in all, selecting at least **ONE** question from each Section. The **FIFTH** question may be attempted from any Section. All questions carry equal marks.

## SECTION—A

1. State and Prove Arzela's Theorem.
2. State and Prove Weierstrass Approximation Theorem.

## SECTION—B

3. What is a measurable function ? Give an example. Also, give an example of a non measurable function and a non-measurable set. Justify each of your claims.
4. State and Prove at least five properties of measurable functions.

## SECTION—C

5. Discuss Littlewood's Three Principles. Explain their significance and prove any result that you use in your discussion.

6. In terms of Simple functions, state and prove a necessary and sufficient condition for a bounded function defined on a set of finite measure to be measurable.

## SECTION—D

7. Prove that an increasing function defined on the closed interval [a, b] is differentiable almost everywhere.

8. What is absolute continuity ? Prove that a function which is absolutely continuous on a closed interval [a, b] is of bounded variations and is differentiable almost everywhere on the interval. Further, show that if its derivative is zero almost everywhere then the function is a constant.